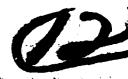


MICROCOPY RESOLUTION TEST CHART NATIONAL BUREAU OF STANDARDS-1963-A



SOUTHERN METHODIST

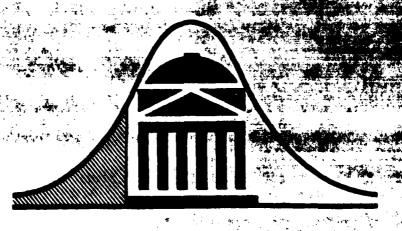
AD A125015

PREDICTION INTERVALS USING EXCEEDANCES FOR AN ADDITIONAL THIRD STAGE SAMPLE

by

D. B. Owen and Youn-Min Chou

Technical Report No. 173 Department of Statistics - ONR Contract February 15, 1983



DEPARTMENT OF STATISTICS

DALLAS, TEXAS 75275

DISTRIBUTION STATEMENT A

Approved for public release;

003 88

PREDICTION INTERVALS USING EXCEEDANCES FOR AN ADDITIONAL THIRD STAGE SAMPLE

by

D. B. Owen and Youn-Min Chou

Technical Report No. 173

Department of Statistics - ONR Contract
February 15, 1983

Research sponsored by the Office of Naval Research Contract N00014-76-C-0613

Reproduction in whole or in part is permitted for any purpose of the United State Government

The document has been approved for public release and sale; its distribution is unlimited

DEPARTMENT OF STATISTICS
Southern Methodist University
Dallas, TX 75275

Accession For					
NTIS	GRALI	X			
DTIC :	LAB				
	ounced				
Just1	Cication	1			
	ibution labilit	y Codes			
	Avail 8	-			
Dist	Speci	al			
1] [
ΙИ	1 1	•			
1 1 /	ł į	, f			



PREDICTION INTERVALS USING EXCEEDANCES FOR AN ADDITIONAL THIRD STAGE SAMPLE

D. B. Owen, Senior Member ASQC Southern Methodist University Dallas, Texas 75275 Youn-Min Chou
The University of Texas at
San Antonio
San Antonio, Texas 78285

Key Words: Normal Prediction Intérvals; Sample Size Tables; Factors for Three-Stage Sampling

Reader Aids-

Purpose: Widen state of the art.

Special math needed for explanations: Probability distributions

Special math needed for results: Probability distributions

Results useful to: Statistically inclined reliability engineers, statisticians

ABSTRACT

Prediction intervals are extended to a third sampling stage involving the distribution of the t-th smallest value in a third stage sample which exceeds the k-th smallest value in the second sample. Procedures and tables are given for two situations. In the first situation the usual 2-stage prediction interval has been applied, and a third stage is now required. Sample sizes are given for this problem. In the second situation we know in advance that three stages will be necessary and the factors are given for the required procedure.

1. INTRODUCTION

In [2] "warranty periods" are given on the future production of m items from an s-normal population, all m of which meet a criterion which is obtained from a preliminary sample of size n. In [3] these calculations are extended so that m-k+1 out of the m future production items are to meet the warranty criterion.

This paper extends these results to a third stage. The secondary production (the future production in [2,3]) met the criterion and we are now required to go into a tertiary stage where ℓ -t+1 out of ℓ additional items are needed to meet a criterion based on the number in the third stage that exceed an order statistic from the second stage. That is, we know that at least m-k+1 out of m items produced at the secondary stage were larger than \overline{X} -rS (where \overline{X} and S were based on a sample of size n at the first stage) and we are required to have ℓ -t+1 out of ℓ items in the third stage larger than the k-th smallest item out of m in the second stage. The third stage must be accomplished with an s-confidence of 1- β , i.e., the probability is 1- β that ℓ -t+1 items manufactured at the third stage will exceed the k-th smallest item at the second stage, given the k-th smallest item at the second stage is larger than \overline{X} -rS.

We will solve this 3-stage problem for two situations.

Model A: The criterion of [2,3] has been applied successfully. Then it is necessary for additional production to meet the criterion that ℓ -t+1 out of ℓ items on a third stage are larger than the k-th smallest item of the second stage. Here we solve for the sample size, ℓ , given the other parameters of the problem.

Model B: We know in advance that we are going to have three stages and in this case we solve for the value of r in the criterion

$$\Pr\{Y_{(k)} < Z_{(t)} | \overline{X} - rS < Y_{(k)}\} = 1 - \beta$$
 (1.1)

Assumptions:

- 1. All observations are from an s-normal distribution with mean μ and variance $\sigma^2.$
- 2. The first stage observations are X_1, X_2, \dots, X_n .
- 3. The second stage observations are Y_1, Y_2, \dots, Y_m .
- 4. The third stage observations are Z_1, Z_2, \ldots, Z_f .
- 5. All of the $n+m+\ell$ observations are s-independent of each other.

Notation list:

 \overline{X} is the sample mean of the first stage sample of size n.

 ${\mbox{S}}^2$ is the sample variance of the first stage sample of size n, i.e.,

$$s^2 \equiv \sum_{i=1}^{n} (x_i - \overline{x})^2 / (n-1).$$

 $Y_{(k)}$ is the k-th smallest observation from the second stage sample of size m.

 $Z_{(t)}$ is the t-th smallest observation from the third stage sample of size ℓ .

 $V_{(k+j)}$ is the (k+j)-th order statistic from a sample of size m+1 from an s-normal distribution with mean μ and variance σ^2 .

 $1-\beta$ is the s-confidence that ℓ -t+1 items manufactured at the third stage will exceed the k-th smallest item at the second stage, given that the k-th smallest item at the second stage is larger than X - rS.

2. THE 3-STAGE JOINT PROBABILITY

The joint probability can be expressed as

$$Pr(\overline{X} - rS < Y_{(k)} < Z_{(t)})$$

$$= \frac{1}{\binom{m+\ell}{\ell}} \int_{j=0}^{t-1} \binom{j+k-1}{j} \binom{m+\ell-k-j}{m-k} \operatorname{Pr}\{\overline{X} - rs < V_{(k+j)}\}$$
 (2.1)

That is, the 3-stage joint probability can be expressed as a finite sum of weighted probabilities computed by the process given in [3].

3. SAMPLE SIZE REQUIRED FOR MODEL A

We assume that the user has conducted the two-stage process as given by Fertig and Mann [3] and that the product at the second stage has met the criterion that the k-th order statistic $Y_{(k)}$ is greater than \overline{X} -rS where r is obtained from tables in [3]. Now we wish to find the sample size, ℓ , required so that $\Pr\{Z_{(t)} > Y_{(k)} \mid Y_{(k)} > \overline{X} - rS\} = 1 - \beta$.

Since it seems reasonable that the same risk be applied to the third stage as was used on the second stage, we also assume that $\Pr\{Y_{(k)} > \overline{X} - rS\} = 1 - \beta$. Then we wish to find the maximum value of ℓ so that $\Pr\{\overline{X} - rS < Y_{(k)} < Z_{(t)}\} \ge (1 - \beta)^2$.

Table 1 gives the sample size, ℓ , needed at the third stage to satisfy this inequality for β = 0.10, 0.05. A dash indicates it is impossible to attain the required value of β .

The first stage sample size, n, has little effect on the probability $\Pr\{\overline{X} - rS < Y_{(k)} < Z_{(t)}\}$. As the sample size n increases from 2 to ∞ , the value of ℓ decreases by at most 3. We therefore printed in Table 1 only the values of ℓ for n=2 and indicate the amount of decrease in ℓ between n=2 and $n=\infty$ by placing *'s after the entry with the number of *'s indicating the amount of the decrease, i.e., * indicates ℓ decreased by one between n=2 and $n=\infty$, ** indicates ℓ decreased by two between n=2 and $n=\infty$, etc.

4. DETERMINATION OF THE FIRST STAGE FACTOR r FOR MODEL B

In the situation considered in this section we assume that we know

we are going to have three stages before the first stage is completed. That is, we want to find a value of r similar to the value of r given by the prediction intervals as presented in [3]. This time, however, we want to find r such $\Pr\{Y_{(k)} < Z_{(t)} \mid \overline{X} - rS < Y_{(k)}\} = 1 - \beta$. Table 2 gives some representative values of r.

5. EXAMPLE

We extend the numerical example given in [3, p. 177]. In that example turbine nozzles made of cast alloy MAR-M 246 cc were to have a constant load of 25K psi. A group of 50 nozzles was available and 10 of them were randomly selected and life-tested in order to predict the failure time of the remaining 40. The sample mean and sample standard deviation of the log failure times for these 10 nozzles were 3.850 and 0.034, respectively. An s-normal distribution for the failure times was assumed. For n = 10, m = 40, k = 5 and k = 0.05, Fertig and Mann [3] found that k = 2.37. Hence a 95 percent lower prediction limit for the time of failure 5 for the remaining 40 nozzles is k = 2.37. k = 2.37. k = 2.37.

For the problem discussed in Model A of this paper, we need additional nozzles for the third stage knowing that among the 40 nozzles of the second stage the fifth failure was after 43.4 hours. Suppose that we want to be 95% sure that the fifth failure time in a third stage sample is larger than the fifth failure time on the second stage, what is the largest sample size we can take on the third stage to meet these conditions? From Table 1 with β = 0.05, n = 10, m = 40, k = 5 = t, we find ℓ = 14. Hence with a sample of size 14 there is 95% s-confidence that at least 10 out of 14 of the third stage sample will have a longer life than the fifth failure time at the second stage.

For an example using the tables of Model B, we will assume that n=10, m=l=40, k=2, t=6 and $\beta=.10$. From Table 2, we get r=1.6521. Hence our criterion for the nozzles of Fertig and Mann's [3] example is $\exp[3.850-(1.6521)(0.034)]=44.4$ hours. Now there is a 90% s-confidence that the sixth failure out of 40 sampled at the third stage will be greater than the second failure at the second stage knowing that the second failure out of 40 sampled at the second failure

ACKNOWLEDGMENTS

The research represented herein was supported by the U.S. Office of Naval Research Contract No. N00014-76-C-0613.

REFERENCES

- [1] Benjamin Epstein. "Tables for the distribution of the number of exceedances," Annals of Mathematical Statistics, 25, 1954, 762-768.
- [2] K. W. Fertig and N. R. Mann. "A new approach to the determination of exact and approximate one-sided prediction intervals for normal and lognormal distributions," <u>Reliability and Fault Tree Analysis</u>, Edited by R. E. Barlow, J. B. Fussell, and N. D. Singpurwalla, Society for Industrial and Applied Mathematics, 1975.
- [3] K. W. Fertig and N. R. Mann. "One-sided prediction intervals for at least pout of m future observations from a normal population,"

 <u>Technometrics</u>, 19, 1977, 167-177.

BIOGRAPHIES

- Dr. Youn-Min Chou; Division of Mathematics, Computer Science and Systems

 Design; The University of Texas, San Antonio, Texas 78285, U.S.A.

 Youn-Min Chou is an Assistant Professor of Statistics at the University

 of Texas at San Antonio. Her research interests include quality con
 trol, estimation and prediction, and distribution theory.
- Dr. D. B. Owen, Department of Statistics, Southern Methodist University,
 Dallas, Texas 75275, U.S.A.
 - D. B. Owen is a professor of statistics at Southern Methodist University. He is an ASQC certified reliability engineer, and operates a consulting service for Don-El Associates, Inc. He is a Fellow of the American Statistical Association, the Institute of Mathematical Statistics, and AAAS. His research interests include screening procedures, quality control, tolerance limits, and sampling plans.

TABLE 1
Sample Sizes Required for Third Stage (Model A)

8	k	t	m→	20	30	40	50	60	70	80
.10	2	2		5*	7	10*	12*	14	17*	19*
		3		10*	14	19*	24*	28*	33**	38**
		4		15*	22*	29*	36*	43**	50**	57 * *
		5		20*	29	39*	48*	58**	68***	3/^^ 77***
	3	2		3	5	6	8	10*	11	13*
		3		7*	10	13	16	2 0*	23*	26*
		4		10	15	20	25*	30*	35*	40*
		5		14	21*	28*	34*	41*	48**	55**
	4	2		2	4*	5	6	7	9*	10*
		3		5	8*	10	13*	15	18*	20*
		4		8	12	16*	20*	23	27*	31*
		5		11	16	22*	27*	32*	37*	43**
	5	2		2	3	4	· 5	6	7	8
		3		4	6	8	10	1.2	14	16
		4		7	10	13	16	19	22	25
		5		9	14*	18*	22*	26	_31*	35*
										
.05	2	2		3	5	6	8	9	11	12
		3		7	10	14*	17	20	23	27*
		4		11	16	22*	27*	32*	37*	42
		5		16*	23*	30*	37*	44*	51	58
	3	2		2	3	4	5	6	7	9*
		3		5	7	10*	12	14	17*	19
		4		8	12	· 16*	19	23	27*	30
		5		11	17*	22*	27*	32	37	42
	4	2		2	2	3	4	5	6	6
		3		4	6	7	9	11	13	15*
		4		6	9	12	15	18	21	24*
		5		9	13	17	21	25	29	33
	5	2		-	2	3	3	4	5*	5
		3		3	5	6	8*	9	11*	12
		4		5	8	10	12	15	17	20*
		5		8		14	18*	21	24	28*

TABLE 2

Factors for obtaining $100(1-\beta)$ % 1 -sided exceedances for containing at least ℓ -t+1 out of ℓ additional observations at a third stage given a first stage sample of size n and a second stage sample of size $m(=\ell)$ for which at least m-k+1 out of m observations were in the prediction interval (Model B).

		\ \ =	1, t= 4,	$\beta = .10$			
l	20	30	40	50	50	7)	30
n 2 4 5 3 10 12 14 15 20 30 40 50 70 30 90	1.3345 1.5499 1.7550 1.3024 1.3274 1.3418 1.3503 1.3503 1.8530 1.8630 1.3633 1.3633 1.3633 1.3635 1.3635 1.3635 1.3635	1.4595 1.3372 1.9595 2.0150 2.0442 2.0510 2.0713 2.0779 2.0321 2.0349 2.0339 2.0377 2.0352 2.0350 2.0339 2.0339 2.0350 2.0329 2.0329	1.5343 1.9425 2.0794 2.1422 2.1755 2.1951 2.2071 2.2147 2.2197 2.2230 2.2283 2.2277 2.2253 2.2243 2.2227 2.2214 2.2202 2.2061	1.5723 2.0113 2.1505 2.2293 2.2571 2.2391 2.3027 2.3116 2.3174 2.3213 2.3280 2.3275 2.3253 2.3253 2.3253 2.3239 2.3212 2.3205 2.3192 2.3031	1.5952 2.0505 2.2203 2.2952 2.3350 2.3755 2.3355 2.3922 2.3957 2.4043 2.4043 2.4031 2.4011 2.3993 2.3975 2.3952 2.3952	1.5124 2.0930 2.2557 2.3467 2.3467 2.4170 2.4337 2.4448 2.4523 2.4574 2.4571 2.4575 2.4559 2.4540 2.4521 2.4533 2.4533 2.4533	1.3241 2.1275 2.3342 2.3337 2.4355 2.4333 2.4319 2.5022 2.5079 2.5126 2.5126 2.5126 2.5136 2.5136 2.5136 2.5136 2.5115 2.4915
		<u>'</u> <= ½	2, t= 6,	β= .10			
l n	50	30	40	50	50	70	30
2 4 5 3 10 12 14 16 18 20 30 40 50 70 90	.5716 .9955 1.1217 1.1853 1.2244 1.2490 1.2659 1.2731 1.2872 1.2872 1.32943 1.3137 1.3222 1.3268 1.3296 1.3315 1.3328 1.3338 1.3396	.8333 1.2417 1.3307 1.4517 1.4934 1.5202 1.5394 1.5513 1.5609 1.5632 1.5935 1.6005 1.6005 1.6025 1.6031 1.6039	.9945 1.3774 1.5233 1.5964 1.6521 1.6313 1.7012 1.7154 1.7253 1.7337 1.7539 1.7614 1.7649 1.7666 1.7675 1.7680 1.7683 1.7659	1.0573 1.4654 1.6275 1.7112 1.7605 1.7924 1.3140 1.3294 1.8403 1.8493 1.8712 1.8792 1.8325 1.8355 1.8355 1.8355	1.0935 1.5273 1.5995 1.7335 1.3413 1.3754 1.8937 1.9153 1.9276 1.9358 1.9505 1.9592 1.9745 1.9753 1.9755 1.9757	1.1275 1.5743 1.7543 1.9435 1.9045 1.9403 1.9657 1.9335 2.0065 2.0321 2.0414 2.0454 2.0472 2.0430 2.0433 2.0403	1.1439 1.5117 1.7390 1.8972 1.9550 1.9943 2.0205 2.0395 2.0534 2.0640 2.0915 2.1015 2.1053 2.1078 2.1039 2.1039

TABLE 2

Pactors for obtaining $100(1-\beta)\%$ l-sided exceedances for containing at least ℓ -t+1 out of ℓ additional observations at a third stage given a first stage sample of size n and a second stage sample of size m(= ℓ) for which at least m-k+1 out of m observations were in the prediction interval (Model B).

2 0 30 40 50 70 70 30 1 .3345		•	:<=	1, t= 4,	8= .10			
1 1.3345 1.4595 1.5343 1.5723 1.5932 1.5124 1.5241 4 1.5499 1.3372 1.9425 2.0113 2.0505 2.0930 2.1275 3 1.7550 1.9595 2.0794 2.1595 2.2203 2.2537 2.3042 3 1.0024 2.0159 2.1422 2.2293 2.2952 2.3467 2.3237 10 1.3274 2.0442 2.1755 2.2571 2.3360 2.3952 2.3467 3.3237 10 1.3274 2.0442 2.1755 2.2571 2.3360 2.3952 2.4355 12 1.8418 2.0510 2.1951 2.2301 2.3503 2.4170 2.4533 14 1.3503 2.0713 2.2071 2.3027 2.3755 2.4237 2.4310 1.3555 2.0779 2.2147 2.3116 2.3355 2.4443 2.4939 18 1.3503 2.0321 2.2197 2.3174 2.3922 2.4523 2.5022 20 1.3530 2.0349 2.2239 2.3116 2.3355 2.4443 2.4939 18 1.3503 2.0349 2.2239 2.3114 2.3922 2.4523 2.5022 20 1.3530 2.0349 2.2239 2.3213 2.3957 2.4574 2.5079 30 1.3530 2.0394 2.2233 2.3200 2.4043 2.4571 2.5191 40 1.2535 2.0339 2.2277 2.3275 2.4043 2.4571 2.5191 40 1.2535 2.0339 2.2277 2.3275 2.4043 2.4575 2.5290 1.3533 2.0377 2.2253 2.3253 2.4031 2.4559 2.5157 50 1.3571 2.0350 2.2227 2.3221 2.3993 2.4521 2.5143 30 1.3555 2.0339 2.2227 2.3221 2.3993 2.4521 2.5143 30 1.3555 2.0339 2.2227 2.3221 2.3993 2.4521 2.5143 30 1.3555 2.0339 2.2227 2.3221 2.3993 2.4521 2.5143 30 1.3555 2.0339 2.2227 2.3221 2.3993 2.4521 2.5143 30 1.3555 2.0339 2.2227 2.3221 2.3993 2.4521 2.5143 30 1.3555 2.0339 2.2214 2.3205 2.3975 2.4503 2.5115 ∞ 1.8593 2.0719 2.2051 2.3031 2.3734 2.4399 2.4015 2.4549 1.3574 1.3774 1.4554 1.5273 1.5743 1.5117 1.3907 1.5233 1.5233 1.5275 1.5955 1.7543 1.5975 1.2447 1.3774 1.4554 1.5273 1.5743 1.5917 1.2444 1.4934 1.5521 1.7506 1.3413 1.9045 1.9570 12 1.2444 1.4934 1.5521 1.7506 1.3413 1.9045 1.9570 12 1.2444 1.4934 1.5521 1.7506 1.3413 1.9045 1.9570 12 1.2444 1.4934 1.5521 1.7506 1.3413 1.9045 1.9570 12 1.2444 1.4934 1.5521 1.7506 1.3413 1.9045 1.9570 12 1.2490 1.5202 1.6313 1.7924 1.8937 1.9595 2.0534 2.0595 1.2417 1.3737 1.8493 1.9575 2.0493 2.0055 2.0540 1.2943 1.5502 1.7539 1.8712 1.9505 2.0321 2.0915 1.2443 1.5502 1.7539 1.7549 1.9950 2.0534 2.0055 1.3265 1.5598 1.7549 1.8351 1.9755 2.0443 2.1059 1.3333 1.5031 1.7533 1.8355 1.9755 2.0443 2.1059 1.3333 1.5031 1.7533 1.8355 1.9757 2.0443 2.105			,,	-,,	, , ,			
1.3345	l	20	30	10	50	50	70	30
4 1.3499 1.3372 1.9425 2.0113 2.0305 2.0030 2.1275 5 1.7550 1.9595 2.0794 2.1505 2.2203 2.2557 2.3042 2.1505 1.9595 2.2737 2.3042 2.1275 1.3024 2.0150 2.1422 2.2293 2.2952 2.3467 2.3337 1.3274 2.0442 2.1753 2.2671 2.3360 2.3905 2.4355 12 1.3418 2.0510 2.1951 2.2331 2.3503 2.4170 2.4533 14 1.3503 2.0713 2.2071 2.3027 2.3755 2.4327 2.4315 1.3503 2.0713 2.2071 2.3027 2.3755 2.4327 2.4315 1.3503 2.0321 2.2197 2.3174 2.3922 2.4523 2.5022 2.1470 2.3530 2.0321 2.2197 2.3174 2.3922 2.4523 2.5022 2.15303 2.0334 2.2233 2.3213 2.3957 2.4443 2.4930 1.3530 2.0349 2.2233 2.3213 2.3957 2.4471 2.5191 4.2535 2.0339 2.2277 2.3275 2.4043 2.4571 2.5191 4.2535 2.0339 2.2277 2.3275 2.4043 2.4575 2.5290 1.3533 2.0377 2.2253 2.3252 2.4043 2.4575 2.5290 1.3537 2.0352 2.2243 2.3239 2.4011 2.4540 2.5157 7.3 1.3571 2.0352 2.2227 2.3212 2.3993 2.4521 2.5143 30 1.3655 2.0339 2.2214 2.3205 2.3993 2.4521 2.5143 30 1.3655 2.0339 2.2214 2.3205 2.3993 2.4521 2.5143 30 1.3655 2.0339 2.2214 2.3205 2.3993 2.4521 2.5143 30 1.3655 2.0339 2.2214 2.3205 2.3993 2.4521 2.5143 30 1.3655 2.0339 2.2214 2.3205 2.3932 2.4533 2.5115 2.515 1.8595 2.0719 2.2051 2.3031 2.3734 2.4390 2.4015 2.4015 2.4016	n					_		
1.7550 1.9595 2.0794 2.1505 2.2203 2.2537 2.3042 3 1.0024 2.0150 2.1422 2.2293 2.2052 2.3467 2.3237 10 1.3274 2.0442 2.1755 2.2571 2.3360 2.3905 2.4355 12 1.3418 2.0510 2.1951 2.2391 2.3303 2.4170 2.4533 14 1.3503 2.0713 2.2071 2.3027 2.3755 2.4337 2.4310 15 1.3555 2.0779 2.2147 2.3116 2.3955 2.4443 2.4930 18 1.3503 2.0321 2.2197 2.3174 2.3922 2.4523 2.5022 20 1.3530 2.0349 2.2230 2.3213 2.3957 2.4574 2.5072 30 1.3530 2.0394 2.2233 2.3230 2.4043 2.4571 2.5191 40 1.2535 2.0339 2.2277 2.3275 2.4043 2.4575 2.5290 50 1.3503 2.0339 2.2277 2.3275 2.4043 2.4575 2.5290 50 1.3563 2.0339 2.2277 2.3275 2.4043 2.4575 2.5290 50 1.3563 2.0339 2.2277 2.3275 2.4043 2.4575 2.5290 51 1.3677 2.0352 2.2243 2.3239 2.4011 2.4559 2.5136 52 1.3677 2.0352 2.2243 2.3239 2.4011 2.4540 2.5157 70 1.3671 2.0350 2.2227 2.3221 2.3993 2.4521 2.5143 30 1.3655 2.0339 2.2214 2.3205 2.3976 2.4503 2.5153 90 1.3550 2.0029 2.2202 2.3192 2.3952 2.4533 2.5115 ∞ 1.8593 2.0719 2.2051 2.3031 2.3734 2.4390 2.4015 ***e 2, t= 5, β = .10 ***e 2, t= 6, β = .10 **								
1.3024 2.0150 2.1422 2.2293 2.2052 2.3467 2.3237 10 1.3274 2.0442 2.1755 2.2571 2.3350 2.3905 2.4355 12 1.8418 2.0510 2.1951 2.2391 2.3303 2.4170 2.4533 14 1.3503 2.0713 2.2071 2.3027 2.3755 2.4337 2.4310 15 1.8555 2.0779 2.2147 2.3116 2.3355 2.4443 2.4930 18 1.3603 2.0321 2.2197 2.3174 2.3922 2.4523 2.5022 20 1.3530 2.0349 2.2230 2.3213 2.3957 2.4574 2.5979 30 1.3630 2.0394 2.2233 2.3280 2.4043 2.4671 2.5191 40 1.2535 2.0339 2.2277 2.3275 2.4043 2.4671 2.5191 41 2.535 2.0339 2.2277 2.3275 2.4043 2.4675 2.5290 1.3673 2.0377 2.2253 2.3253 2.4043 2.4675 2.5290 1.3677 2.0352 2.2243 2.3239 2.4011 2.4659 2.5167 7.0 1.3671 2.0350 2.2227 2.3221 2.3993 2.4521 2.5143 30 1.3655 2.0339 2.2214 2.3205 2.3975 2.4633 2.5130 90 1.3550 2.0329 2.2214 2.3205 2.3975 2.4633 2.5130 90 1.3550 2.0029 2.2202 2.3192 2.3952 2.4533 2.5130 90 1.3655 2.0339 2.2214 2.3205 2.3975 2.4633 2.5130 90 1.3655 2.0339 2.2214 2.3205 2.3975 2.4633 2.5130 90 1.3655 2.0339 2.2214 2.3205 2.3975 2.4633 2.5130 90 1.3655 2.0399 2.2202 2.3192 2.3952 2.4533 2.5115 ∞ 1.8593 2.0719 2.2051 2.3031 2.3734 2.4390 2.4915 c= 2, k= 5, β= .10 2 20 30 40 50 50 70 30 1 1.853 1.4517 1.5064 1.7112 1.7335 1.2435 1.9916 1.2244 1.4334 1.6521 1.7506 1.3413 1.9046 1.9560 12 1.2249 1.5202 1.6313 1.7924 1.8754 1.9408 1.9943 14 1.2659 1.5334 1.7012 1.3140 1.8937 1.9557 2.0205 16 1.2731 1.5513 1.7154 1.8294 1.9153 1.9355 2.0395 13 1.2372 1.5509 1.7253 1.8408 1.9276 1.9965 2.0534 1 1.2943 1.5592 1.7337 1.8493 1.9359 2.0055 2.0540 30 1.3137 1.5373 1.7539 1.8712 1.9506 2.0321 2.0915 50 1.3263 1.5945 1.7549 1.8326 1.9733 2.0452 2.0534 50 1.3263 1.5995 1.7549 1.8325 1.9753 2.0493 2.1099 1.3338 1.6005 1.7545 1.8351 1.9753 2.0493 2.1099 1.3338 1.6005 1.7565 1.8343 1.9745 2.0472 2.1073 70 1.3315 1.6018 1.7575 1.8351 1.9757 2.0493 2.1099								
10								
12								
14								
16								
18								
20								
1.3530								
40								
1.5577 2.0352 2.2243 2.3239 2.4011 2.4540 2.5157 70 1.3671 2.0350 2.2227 2.3221 2.3993 2.4521 2.5143 30 1.3655 2.0339 2.2214 2.3205 2.3975 2.4503 2.5130 90 1.3650 2.0329 2.2202 2.3192 2.3952 2.4533 2.5115			2.0339	2.2277	2.3275	2.4043	2.4375	2.5290
73	50		2.0377					
1.3655 2.0339 2.2214 2.3205 2.3975 2.4503 2.5130 90 1.3650 2.0329 2.2202 2.3192 2.3932 2.4533 2.5115 1.8596 2.0719 2.2051 2.3031 2.3734 2.4399 2.4915								
90								
\$\begin{array}{c c c c c c c c c c c c c c c c c c c								
k 20 30 40 50 50 70 30 2 .5715 .3383 .9945 1.0573 1.0035 1.1275 1.1430 4 .9955 1.2417 1.3774 1.4554 1.5273 1.5743 1.5117 5 1.1217 1.3307 1.5233 1.5275 1.5995 1.7543 1.7990 3 1.1853 1.4517 1.5064 1.7112 1.7335 1.3435 1.3972 10 1.2244 1.4934 1.6521 1.7506 1.3413 1.9045 1.9550 12 1.2490 1.5202 1.6313 1.7924 1.9754 1.9408 1.9943 14 1.2659 1.5394 1.7012 1.8140 1.8937 1.9557 2.0205 16 1.2781 1.5513 1.7154 1.8294 1.9153 1.9335 2.0395 13 1.2372 1.5509 1.7253 1.8408 1.9276 1.9956 2.0534 20 1.2943 1.5632 1.7337 1.8493 1.9363 2.0055 2.0540 30 1.3137 1.5373 1.7539 1.8712 1.9506 2.0321 2.0915 40 1.3222 1.5948 1.7514 1.8792 1.9506 2.0321 2.0915 50 1.3263 1.5985 1.7549 1.8325 1.9723 2.0454 2.1058 50 1.3296 1.6005 1.7656 1.8343 1.9745 2.0472 2.1078 70 1.3315 1.6018 1.7575 1.8351 1.9753 2.0490 2.1035 80 1.3328 1.6026 1.7630 1.8355 1.9757 2.0493 2.1089 90 1.3333 1.5031 1.7633 1.8355 1.9757 2.0493 2.1090								
g 20 30 40 50 50 70 30 n .5715 .3383 .9945 1.0573 1.0235 1.1275 1.1430 4 .9955 1.2417 1.3774 1.4654 1.5273 1.5743 1.6117 5 1.1217 1.3307 1.5233 1.5275 1.5995 1.7543 1.7920 3 1.1863 1.4517 1.6064 1.7112 1.7335 1.3435 1.3972 10 1.2244 1.4934 1.6521 1.7505 1.3413 1.9045 1.9560 12 1.2490 1.5202 1.6313 1.7924 1.3754 1.9408 1.9943 14 1.2659 1.5394 1.7012 1.8140 1.8937 1.9557 2.0205 16 1.2731 1.5513 1.7154 1.8294 1.9153 1.9335 2.0353 20 1.2372 1.5509 1.7253 1.8408 1.9276 1.9955 2.0534 20 1.2943 1.5682 1.7337 1.8493 1.9368 2.0055 2.0540 30 1.3137 1.5373 1.7549 1.83712 1.9606 2.0321 2.0915 40 <	00	1.8595	2.0/19	2.2031	2.3031	2.3/34	2.4399	2.4715
g 20 30 40 50 50 70 30 n .5715 .3383 .9945 1.0573 1.0035 1.1275 1.1430 4 .9955 1.2417 1.3774 1.4554 1.5273 1.5743 1.6117 5 1.1217 1.3307 1.5233 1.5275 1.5995 1.7543 1.7930 3 1.1863 1.4517 1.6064 1.7112 1.7335 1.3435 1.3972 10 1.2244 1.4934 1.6521 1.7505 1.3413 1.9045 1.9560 12 1.2490 1.5202 1.6313 1.7924 1.3754 1.9408 1.9943 14 1.2659 1.5394 1.7012 1.8140 1.8937 1.9557 2.0205 16 1.2731 1.5513 1.7154 1.8294 1.9153 1.9335 2.0395 13 1.2372 1.5509 1.7253 1.8408 1.9276 1.9955 2.0534 20 1.2943 1.5632 1.7337 1.8493 1.9359 2.0055 2.0540 30 1.3137 1.5373 1.7539 1.8712 1.9506 2.0321 2.0915 40 <t< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<>								
g 20 30 40 50 50 70 30 n .5715 .3383 .9945 1.0573 1.0035 1.1275 1.1430 4 .9955 1.2417 1.3774 1.4554 1.5273 1.5743 1.6117 5 1.1217 1.3307 1.5233 1.5275 1.5995 1.7543 1.7930 3 1.1863 1.4517 1.6064 1.7112 1.7335 1.3435 1.3972 10 1.2244 1.4934 1.6521 1.7505 1.3413 1.9045 1.9560 12 1.2490 1.5202 1.6313 1.7924 1.3754 1.9408 1.9943 14 1.2659 1.5394 1.7012 1.8140 1.8937 1.9557 2.0205 16 1.2731 1.5513 1.7154 1.8294 1.9153 1.9335 2.0395 13 1.2372 1.5509 1.7253 1.8408 1.9276 1.9955 2.0534 20 1.2943 1.5632 1.7337 1.8493 1.9359 2.0055 2.0540 30 1.3137 1.5373 1.7539 1.8712 1.9506 2.0321 2.0915 40 <t< td=""><td></td><td></td><td>ζ=</td><td>2. t = 5.</td><td>$\beta = .10$</td><td></td><td></td><td></td></t<>			ζ=	2. t = 5.	$\beta = .10$			
0 .5716 .3383 .9945 1.0573 1.0035 1.1275 1.1430 4 .9955 1.2417 1.3774 1.4554 1.5273 1.5743 1.5117 5 1.1217 1.3907 1.5233 1.5275 1.5995 1.7543 1.7990 3 1.1853 1.4517 1.5064 1.7112 1.7335 1.3435 1.3972 10 1.2244 1.4934 1.6521 1.7505 1.3413 1.9045 1.9550 12 1.2490 1.5202 1.6313 1.7924 1.3754 1.9408 1.9943 14 1.2659 1.5384 1.7012 1.8140 1.8937 1.9657 2.0205 16 1.2731 1.5513 1.7154 1.8294 1.9153 1.9355 2.0395 13 1.2372 1.5509 1.7253 1.8408 1.9276 1.9965 2.0534 20 1.2943 1.5632 1.7337 1.8493 1.9369 2.0055 2.0540 30 1.3137 1.5373 1.7539 1.8712 1.9606<				•	•			
2 .5716 .3383 .9945 1.0573 1.0235 1.1275 1.1432 4 .9955 1.2417 1.3774 1.4554 1.5273 1.5743 1.6117 5 1.1217 1.3307 1.5233 1.5275 1.5995 1.7543 1.7920 3 1.1863 1.4517 1.5064 1.7112 1.7335 1.3435 1.3972 10 1.2244 1.4934 1.6521 1.7506 1.3413 1.9045 1.9560 12 1.2490 1.5202 1.6313 1.7924 1.3754 1.9408 1.9943 14 1.2659 1.5384 1.7012 1.8140 1.8937 1.9557 2.0205 16 1.2731 1.5513 1.7154 1.8294 1.9153 1.9335 2.0395 13 1.2372 1.5509 1.7253 1.8408 1.9276 1.9956 2.0534 20 1.3137 1.5373 1.7539 1.8712 1.9606 2.0321 2.0915 40 1.3222 1.5948 1.7514 1.8792 1.9592<	L	20	30	40	50	50	70	30
4 .9955 1.2417 1.3774 1.4554 1.5273 1.5743 1.6117 5 1.1217 1.3807 1.5233 1.5275 1.5995 1.7543 1.7920 3 1.1853 1.4517 1.6064 1.7112 1.7335 1.3435 1.3972 10 1.2244 1.4934 1.6521 1.7505 1.3413 1.9045 1.9560 12 1.2490 1.5202 1.6313 1.7924 1.8754 1.9408 1.9943 14 1.2659 1.5384 1.7012 1.8140 1.8937 1.9557 2.0205 16 1.2781 1.5513 1.7154 1.8294 1.9153 1.9335 2.0395 13 1.2372 1.5509 1.7253 1.8408 1.9276 1.9955 2.0534 20 1.2943 1.5632 1.7337 1.8493 1.9363 2.0055 2.0540 30 1.3137 1.5373 1.7539 1.8712 1.9606 2.0321 2.0915 40 1.3222 1.5943 1.7514 1.8792 1.9								
1.1217 1.3307 1.5233 1.5275 1.5995 1.7543 1.7990 3 1.1853 1.4517 1.6064 1.7112 1.7335 1.3485 1.3972 10 1.2244 1.4934 1.6521 1.7505 1.3413 1.9045 1.9550 12 1.2490 1.5202 1.6313 1.7924 1.3754 1.9408 1.9943 14 1.2659 1.5384 1.7012 1.8140 1.8937 1.9657 2.0205 16 1.2731 1.5513 1.7154 1.8294 1.9153 1.9335 2.0395 13 1.2372 1.5509 1.7253 1.8408 1.9276 1.9965 2.0534 20 1.2943 1.5692 1.7337 1.8493 1.9369 2.0055 2.0540 30 1.3137 1.5373 1.7539 1.8712 1.9506 2.0321 2.0915 40 1.3222 1.5949 1.7514 1.8792 1.9592 2.0414 2.1015 50 1.3263 1.5985 1.7549 1.8343 1.9745								
3 1.1853 1.4517 1.5064 1.7112 1.7335 1.3435 1.3972 10 1.2244 1.4934 1.6521 1.7506 1.3413 1.9045 1.9560 12 1.2490 1.5202 1.6313 1.7924 1.3754 1.9408 1.9943 14 1.2659 1.5384 1.7012 1.3140 1.8937 1.9557 2.0205 16 1.2731 1.5513 1.7154 1.8294 1.9153 1.9335 2.0395 13 1.2372 1.5509 1.7253 1.8408 1.9276 1.9956 2.0534 20 1.2943 1.5632 1.7337 1.8493 1.9363 2.0055 2.0540 30 1.3137 1.5373 1.7539 1.8712 1.9606 2.0321 2.0915 40 1.3222 1.5948 1.7614 1.8792 1.9592 2.0414 2.1015 50 1.3263 1.5985 1.7549 1.8325 1.9723 2.0454 2.1059 60 1.3295 1.6005 1.7565 1.8343	4							
10 1.2244 1.4934 1.6521 1.7506 1.3413 1.9046 1.9560 12 1.2490 1.5202 1.6313 1.7924 1.3754 1.9408 1.9943 14 1.2659 1.5384 1.7012 1.8140 1.8937 1.9557 2.0205 16 1.2731 1.5513 1.7154 1.8294 1.9153 1.9335 2.0395 13 1.2372 1.5509 1.7253 1.8408 1.9276 1.9956 2.0534 20 1.2943 1.5632 1.7337 1.8493 1.9363 2.0055 2.0540 30 1.3137 1.5373 1.7539 1.8712 1.9606 2.0321 2.0915 40 1.3222 1.5948 1.7614 1.8792 1.9592 2.0414 2.1015 50 1.3263 1.5985 1.7549 1.8325 1.9723 2.0454 2.1059 60 1.3295 1.6005 1.7565 1.8343 1.9745 2.0472 2.1073 70 1.3315 1.6013 1.7575 1.8351 <td< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></td<>								
12 1.2490 1.5202 1.6313 1.7924 1.3754 1.9408 1.9943 14 1.2659 1.5384 1.7012 1.8140 1.8937 1.9557 2.0205 16 1.2731 1.5513 1.7154 1.8294 1.9153 1.9335 2.0395 13 1.2372 1.5609 1.7253 1.8408 1.9276 1.9955 2.0534 20 1.2943 1.5632 1.7337 1.8493 1.9363 2.0055 2.0540 30 1.3137 1.5373 1.7539 1.8712 1.9606 2.0321 2.0915 40 1.3222 1.5943 1.7514 1.8792 1.9592 2.0414 2.1015 50 1.3263 1.5985 1.7549 1.8325 1.9723 2.0454 2.1059 50 1.3295 1.6005 1.7565 1.8343 1.9745 2.0472 2.1073 70 1.3315 1.6013 1.7575 1.8351 1.9753 2.0430 2.1035 80 1.3328 1.6026 1.7530 1.8355 <td< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></td<>								
14 1.2659 1.5384 1.7012 1.8140 1.8937 1.9557 2.0205 16 1.2781 1.5513 1.7154 1.8294 1.9153 1.9335 2.0395 13 1.2372 1.5609 1.7253 1.8408 1.9276 1.9956 2.0534 20 1.2943 1.5632 1.7337 1.8493 1.9369 2.0055 2.0540 30 1.3137 1.5373 1.7539 1.8712 1.9506 2.0321 2.0915 40 1.3222 1.5943 1.7614 1.8792 1.9592 2.0414 2.1015 50 1.3263 1.5985 1.7549 1.8325 1.9723 2.0454 2.1059 50 1.3295 1.6005 1.7565 1.8343 1.9745 2.0472 2.1073 70 1.3315 1.6013 1.7575 1.8351 1.9753 2.0490 2.1035 80 1.3328 1.6026 1.7530 1.8355 1.9757 2.0493 2.1090 90 1.3333 1.5031 1.7533 1.8355 <td< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></td<>								
16 1.2731 1.5513 1.7154 1.8294 1.9153 1.9335 2.0395 13 1.2372 1.5509 1.7253 1.8408 1.9276 1.9955 2.0534 20 1.2943 1.5632 1.7337 1.8493 1.9369 2.0055 2.0540 30 1.3137 1.5373 1.7539 1.8712 1.9506 2.0321 2.0915 40 1.3222 1.5948 1.7514 1.8792 1.9592 2.0414 2.1015 50 1.3263 1.5935 1.7549 1.8325 1.9723 2.0454 2.1059 50 1.3295 1.6005 1.7565 1.8343 1.9745 2.0472 2.1073 70 1.3315 1.6013 1.7575 1.8351 1.9753 2.0490 2.1035 80 1.3328 1.6026 1.7530 1.8355 1.9757 2.0493 2.1089 90 1.3333 1.5031 1.7533 1.8355 1.9757 2.0493 2.1090								
13 1.2372 1.5509 1.7253 1.8408 1.9276 1.9955 2.0534 20 1.2943 1.5632 1.7337 1.8493 1.9369 2.0055 2.0540 30 1.3137 1.5373 1.7539 1.8712 1.9506 2.0321 2.0915 40 1.3222 1.5948 1.7514 1.8792 1.9592 2.0414 2.1015 50 1.3263 1.5985 1.7549 1.8325 1.9723 2.0454 2.1059 50 1.3295 1.6005 1.7656 1.8343 1.9745 2.0472 2.1078 70 1.3315 1.6013 1.7575 1.8351 1.9753 2.0430 2.1036 80 1.3328 1.6026 1.7530 1.8355 1.9756 2.0493 2.1089 90 1.3333 1.5031 1.7533 1.8355 1.9757 2.0433 2.1090								2.0395
30 1.3137 1.5373 1.7539 1.8712 1.9506 2.0321 2.0915 40 1.3222 1.5948 1.7514 1.8792 1.9592 2.0414 2.1015 50 1.3263 1.5985 1.7549 1.8325 1.9723 2.0454 2.1059 50 1.3295 1.6005 1.7555 1.8343 1.9745 2.0472 2.1073 70 1.3315 1.6013 1.7575 1.8351 1.9753 2.0430 2.1035 80 1.3328 1.6026 1.7530 1.8355 1.9755 2.0493 2.1089 90 1.3333 1.5031 1.7533 1.8355 1.9757 2.0433 2.1090	13			1.7253	1.8403	1.9276	1.9955	
40 1.3222 1.5948 1.7514 1.8792 1.9592 2.0414 2.1015 50 1.3263 1.5985 1.7549 1.8325 1.9723 2.0454 2.1059 50 1.3295 1.6005 1.7555 1.8343 1.9745 2.0472 2.1073 70 1.3315 1.6013 1.7575 1.8351 1.9753 2.0430 2.1035 80 1.3328 1.6026 1.7530 1.8355 1.9755 2.0493 2.1089 90 1.3333 1.5031 1.7533 1.8355 1.9757 2.0433 2.1090	20	1.2943						
50 1.3263 1.5985 1.7549 1.8325 1.9723 2.0454 2.1059 50 1.3295 1.6005 1.7555 1.8343 1.9745 2.0472 2.1078 70 1.3315 1.6013 1.7575 1.8351 1.9753 2.0490 2.1035 80 1.3328 1.6026 1.7530 1.8355 1.9755 2.0493 2.1089 90 1.3333 1.5031 1.7533 1.8355 1.9757 2.0433 2.1090	30	1.3137	1.5373	1.7539				
50 1.3295 1.6005 1.7565 1.8343 1.9745 2.0472 2.1073 70 1.3315 1.6013 1.7575 1.8351 1.9753 2.0430 2.1035 80 1.3328 1.6026 1.7530 1.8355 1.9755 2.0493 2.1089 90 1.3333 1.5031 1.7533 1.8355 1.9757 2.0493 2.1090								
70 1.3315 1.6018 1.7575 1.8351 1.9753 2.0490 2.1035 80 1.3328 1.6026 1.7530 1.8355 1.9755 2.0493 2.1089 90 1.3333 1.5031 1.7533 1.8355 1.9757 2.0493 2.1090						1.9723		
30 1.3328 1.6026 1.7530 1.8355 1.9755 2.0493 2.1089 90 1.3333 1.5031 1.7533 1.8355 1.9757 2.0493 2.1090								
90 1.3333 1.5031 1.7533 1.8355 1.9757 2.0433 2.1090								

REPORT DOCUMENTATIO	READ INSTRUCTIONS BEFORE COMPLETING FORM	
I. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
173	AD-A1250	り´
4. TITLE (and Subtitle) PREDICTION INTERVALS USING EXCE	EDANCES FOR	S. TYPE OF REPORT & PERIOD COVERED
AN ADDITIONAL THIRD STAGE SAMPL	E.	Technical Report
		6. PERFORMING ORG. REPORT NUMBER
		173
7. AUTHOR(e)		8. CONTRACT OR GRANT NUMBER(s)
D. B. Owen and Youn-Min Chou		N00014-76-C-0613
9. PERFORMING ORGANIZATION NAME AND ADDRI Southern Methodist University Dallas, TX 75275	£3 5	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE
Office of Naval Research		Feb. 15, 1983
Department of the Navy		13. NUMBER OF PAGES
Arlington, VA 22217		10
14. MONITORING AGENCY NAME & ADDRESS(II dille	erent from Controlling Office)	18. SECURITY CLASS. (of this report)
		154. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) This document has been approved	for public releas	se; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

Approved for public release; distribution unlimited.

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Normal Prediction Intervals; Sample Size Tables; Factors for Three-Stage Sampling.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Prediction intervals are extended to a third sampling stage involving the distribution of the t-th smallest value in a third stage sample which exceeds the k-th smallest value in the second sample. Procedures and tables are given for two situations. In the first situation the usual 2-stage prediction interval has been applied, and a third stage is now required. Sample sizes are given for this problem. In the second situation we know in advance that three stages will be necessary and the factors are given for the required

orocedure FORM DD 1 JAN 73 1473

EDITION OF ! NOV 65 IS OBSOLETE S/N 0102-014-6601 |

SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered)

The state of the s